Optimizing Montana’s model to permit urban drainage in humid tropical environment: the case of Abidjan (Côte d’Ivoire)

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ABSTRACT
In sub-Saharan Africa, the "hygienic" model used in urban drainage is faced with constraints in humid tropical environment, subjected to a high level of climatic variability. In Côte d’Ivoire adapting Montana’s rain model is not satisfactory for certain time slots (Peuch and Gonni, 1984) whereas this model includes the Caquot’s rate-of-flow model used in urban drainage. This work aims at optimizing and designing models which best simulate tropical downpours and help in calculations relating to urban drainage in Abidjan and elsewhere. The methodology based on a probabilistic and stochastic approach used rainfall data covering the period from 1958 to 2001, from the Abidjan Airport Weather Station. The various statistical processing, sustained by Kolmogorov and Cramer fit tests showed that the rainfall distribution in Abidjan area is in line with Gumbel’s law (10-30mn rainfall) and that of Galton (45-240mn rainfall). Including the data in mathematical conversion formulas made it possible to optimize the Montana’s parameters and to design a new model which best simulates downpours in Abidjan. In addition, analysis of the Nicholson index reveals a drop in rainfall in Abidjan with a cyclical evolution (17-year period), alternating dry, normal and wet periods. Waiting for its application to judge its value, this new model is relevant since it overlaps two very satisfactory models.

KEYWORDS
Abidjan; climatic variability; humid tropical environment; Montana’s model; urban drainage

INTRODUCTION
Converting rainfall into drainage flow rate is one of the major difficulties faced by hydrologists, city planners and drainage experts in urban centers (Afouda, 1980). The importance of the city of Abidjan, firstly to Côte d’Ivoire and then to the West African sub-region, considerably increased during recent decades. As a trade and business centre in West Africa (Dongo, 2001), Abidjan has become an international economic transit point. The rapid urbanization (42.5\% of the city (INS, 2000) and the lack of an adequate drainage system coupled with variations in the hydrological cycle relating to an insufficiency of urban infrastructures, favour the stagnation of rainwater and floods in Abidjan. So high is the atmospheric precipitations (1800 mm/year) with exceptional rainfalls (heavy and continuous) that the absence of an adequate urban drainage system results not only in temporary floodings
but also in the destruction of roads and dwellings (Paquier et al, 2003). Modelling the hydrological behaviour of drainage basins is very important for solving problems related to the evaluation and optimal management of water resources (Touaizi and Laborde, 2004). Inadequate wastewater and stormwater management results in poor public health, loss of economic productivity and environmental degradation particularly in complex humid environment (Giraud et al, 1997). Since 1995, several districts of Abidjan have suffered a series of floods and grounds instability which had very serious socio-economic consequences (Boyonssoro, 2001). As an example, in 1998, flooding accompanied by a landslide due to heavy and continuous rainfall resulted in the loss of human lives (5 deaths) and several injuries in the district of Abobo. Flood risk mitigation requires a good knowledge of hydrological flood regime (Javelle et al, 2000). These facts raise anxiety within the population and are of concern to both the administrative authorities as well as researchers. It therefore becomes urgent to propose measures capable of preventing such consequences and establishing a drainage program which makes it possible to create harmony between man and the complex system, which makes up his physical, natural environment. However, the success of any action in this area depends on the availability of sufficient information, which allows the consequent application of statistical and probabilistic methods. In sub-Saharan Africa and especially in Côte d’Ivoire, the hygienic concept of rainfall drainage most often used in urban drainage is faced with a certain number of constraints: (i) the Montana's rainfall model is not satisfactory (Puech and Gonni in 1984) and underestimates the intensity of rainfalls, (ii) the Caquot's rate-of-flow model used in urban drainage, includes Montana's model, (iii) adapting parameters of these models to tropical realities was based on the analysis of a limited amount of data from limited areas for a short observation period. A study aimed at bringing these models up to date deserves being carried out in order to allow them to be optimally used for urban drainage. This work aims at optimizing and designing models, which best simulate tropical precipitations and help in calculations for urban drainage in Abidjan and elsewhere. Data from a long series of pluviometric observations will be included in a statistical and stochastic procedure at various levels. Climatic variability in the area will be evaluated by analysing Nicholson index. After optimizing Montana's parameters, which are currently used in pluviometric simulation, we’ll try to propose a rainfall model which takes into account climatic variability in the area.

**METHODS**

**The Montana’s rainfall model and its application**

Research on the estimation of extreme precipitation events is currently expanding (St-Hilaire et al, 2003). Achievements carried out in the field of pluviometric modelling use several models including selective models involving isolated events (Morel, 1996). IDF Curves show the evolution of the maximum intensity ($I_{\text{max}}$) of rain in its natural state. Montana’s formula is in line with this underlying principle for the simulation of rainfall dynamics by the relationship:

$$I_{\text{max}} (T; t) = a(T) \times t^{-b(T)}$$

(1)

with $t$ (the time during which the intensity $I_{\text{max}}$ is reached on the average), $T$ (rainfall return period), $a(T)$ and $b(T)$ (coefficients of adjustment). Thus, for a given area, Montana’s equation is reliable for the parameters $a(T)$ and $b(T)$ evaluated under satisfactory statistical conditions. In the drainage projects of the city of Abidjan, the parameters used are those defined by the Department of Water of the Ministry of public works and Transport in 1986.
Even some researchers (Sigomnou and Desbordes, 1988) tried to find out models to simulate rainfall dynamic in Côte d’Ivoire. All these values were obtained from short-term simulations.

The major advantage of Montana’s formula is that it can easily be represented on a graph. This is of great interest to drainage experts when it comes to estimating the intensity of rainfall. However, the parameters \( a(T) \) and \( b(T) \) greatly depend on the quality of the recording of the variable “rain”, the size of the sample of the data collected, the geographical nature of their recordings and the climatic particularity of the area of data collection. Applying such a relationship to other zones can thus lead to false conclusions (OMS, 1983). Montana’s formula is applied in the Caquot’s rate-of-flow forecast model where it comes in as essential input data. The literal expression of the rate of flow from an urbanised sloppy basin for a given frequency "F" established by Caquot’s work is in the following form:

\[
Q_{\text{max}}(T; t) = \frac{10^3 C_r A^{1-e}}{6(\beta + \lambda)} \times a(T) \times t^{-b(T)}
\]

with \( Q_{\text{max}}(t) \) (the maximum rate of flow at the mouth of the basin), \( A \) (the surface area), \( I \) (the average slope), \( C_r \) (the runoff percentage), \( \beta + \lambda \) (the storage and crest lowering effect). The tests of Caquot’s model’s adaptation to tropical realities (Lemoine and Cruette, 1974) show that the numerical coefficient used in formula (2) vary from one basin to another. However, these shortcomings should not make us to lose sight of the importance of the parameters \( a(T) \) and \( b(T) \) because in formula (2) the various parameters are functions of \( a(T) \) and or \( b(T) \) which are themselves parameters in Montana’s formula. This is therefore logical, as we seek to find solutions aimed at correcting the existing shortcomings, to carry out the optimization of these models which are still used in humid tropical environment and singularly in Abidjan.

Means of optimizing the models

Climatic variability analysis methods Climatic variability at Abidjan Weather Station was studied by NICHOLSON pluvial index method and statistical analysis of rainfall data. Data used were provided by SODEXAM (Airport and Weather Exploitation and Development Company) and cover a 65-year period (1936-2000). This station, the only to have constant data covering a long period of observation, was considered to be representative of the Abidjan area. Moreover, the observation period (65ans) easily meets the requirements of WMO (World Meteorology Organization) directives which recommend an observation period of at least 30 years. Abidjan Station pluviometric fluctuations were calculated based on the Nicholson relationship:

\[
I_p = \frac{R_{ij} - R_{im}}{\sigma}
\]

Rij (the total rainfall level for a season i and a year j); R_{im} (the average annual rainfall of the season i during the duration of recording); \( \sigma \) (the standard fluctuation in annual rainfall). The average rainfall for each decade of the observation period was then used to assess the long term variability of rainfall in Abidjan area.

Optimizing the models When local information on streamflows is insufficient for estimating flood quantiles, a regional flood frequency analysis is usually carried out (Mic et al 2002). In the same way, in the absence of several stations of measures, data from Abidjan-airport station are used to be representative for the whole Abidjan region. The optimization process consists of a series of statistical processing of rainfall data collected at Abidjan Station. This process begins with the examination of available pluviometric diagrams from the 1958-2001 period for a span of time of 10, 15, 30, 45, 60, 90, 120, 180 and 240 mn. The operation
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consists in finding the maximum rainfall intensities or rare events (Garrido, 2002) from the diagrams (pluviometers) for the various time spaces on each graph for the year considered. Of all rainfall level values obtained, the highest value is selected (statistics of placement). The process is repeated for the specified time spaces and for each year of the observation period. Data (rainfall level) obtained are converted into rainfall intensity based on the following formula: 

\[ i = \frac{H}{t} \]  

(4) and recorded in a table.

Examining the data is a delicate task which requires dexterity on the part of the operator in order to have accurate data. A quality control also consisted in an execution by two different operators. Differences in results are re-examined and corrected after comparison. The second stage consists in determining the statistical distribution law of the variables. The data are thus subjected to the most commonly used statistical distribution laws in urban hydrology (Gumbel, Galton and Pearson III). Two statistical fit tests (Kolmogorov and Cramer) believed to be "more powerful" were then applied in order to evaluate the law(s) which best simulate(s) the distribution of the variables. The Kolmogorov test takes into account all the “fractiles” and consists in measuring the difference between the exact distribution function and the empirical distribution function and comparing it to an "acceptable" value. Cramer-Von Mises’ test also aims at determining the law which best expresses the parent population of which the sample is a priori representative. The difference between the two tests is in the fact that the Kolmogorov’s test is more sensitive to the existence of aberrant points in the sample than that of Cramer-Von Mises. The intensity values obtained from the distribution laws applied are included in mathematical simulations. The Excel software is used to create this simulation and the values of correlation are used as indicators for measuring the accuracy of the simulation.

RESULTS

Climatic variability analysis by the Nicholson and statistical methods

Calculating Nicholson index made it possible to evaluate the various climatic fluctuations from 1936 to 2000 and to determine a normal, a wet and two dry periods (figure 1).

![Figure 1. Variation of Nicholson index](image)

From 1966 to 1982 regarded as a normal period, the fluctuation tends to be balanced on both sides of the axis. This phenomenon is observed not only by means of the Nicholson indexes variation graphs but also by the fact that approximately 53% of rainfall totals are higher than the average, which is 1910 mm. This normal period experienced a few short dry periods, the most serious being between 1968 and 1973. From 1983 to 2000, more than 89% of the rainfall totals remain lower than the average, confirming the existence of the second dry period. By establishing a correlation between this graph and the major droughts of West Africa, it can
easily be noticed that those of 1947-1953, 1968-1973 and 1982-1984 mentioned by Biémi (1992) can be seen. Abidjan Station is thus well included in the major fluctuations of the West African climate. In addition, the pluvial phenomenon chronological evolution in Abidjan is similar to that of a cyclical phenomenon of an estimated period of 17 years.

The importance of the climatic variability in Abidjan is highlighted in figure 2. There is a high variability between average decennial rainfalls. As recent studies mentioned it, no single conclusion is valid concerning fluctuations of rainfall in Abidjan area (Biot et al, 2005). For example, average decennial rainfall for 1941-1951 and 1971-1980 are about the same, but both are far less than the average rainfall for 1951-1960 and 1981-1990 period.

Figure 2. Decennial rainfall in relation to the average of the observation period

**Optimizing Montana’s rainfall model parameters**

Examining the Abidjan Station diagrams made it possible to isolate the maximum rainfall levels. The intensity records studied for various time periods show that a heavy and continuous rain that fell in the Abidjan area on June 14, 1961 has never reoccurred up to the present (table1).

<table>
<thead>
<tr>
<th>Duration (mn)</th>
<th>Intensity (mm/h)</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>336.00</td>
<td>June 22 1971</td>
</tr>
<tr>
<td>15</td>
<td>316.40</td>
<td>October, 16 1973</td>
</tr>
<tr>
<td>30</td>
<td>202.00</td>
<td>October, 161973</td>
</tr>
<tr>
<td>45</td>
<td>136.00</td>
<td>May,21 1969</td>
</tr>
<tr>
<td><strong>60</strong></td>
<td><strong>104.00</strong></td>
<td><strong>June, 14 1961</strong></td>
</tr>
<tr>
<td>90</td>
<td>86.67</td>
<td>June, 14 1961</td>
</tr>
<tr>
<td>120</td>
<td>78.00</td>
<td>June, 14 1961</td>
</tr>
<tr>
<td>180</td>
<td>61.00</td>
<td>June, 14 1961</td>
</tr>
<tr>
<td>240</td>
<td>47.13</td>
<td>June, 14 1961</td>
</tr>
</tbody>
</table>

During the observation period (1958-2001), the highest rainfall can be observed in the last quarter of the dry year of 1973, thus confirming the intense character of the periods of precipitation, which come after the dry periods in the Abidjan area. Rainfall data obtained from studies and taken into consideration in the various statistical processing show that short rainfalls (10-30 mn) preferentially obey the Gumbel law while the distribution of long
downpours is more connected with Galton law (45-240 min). Rainfall intensities were therefore estimated based on selected distribution laws. Two groups of curves indicating the distribution of rainfall intensities based respectively on the duration of the downpour and the return period were drawn thanks to the Excel spreadsheet (figure 3 and 4).

**Figure 3.** Evolution of rainfall intensity with respect to the duration of downpours in Abidjan

**Figure 4.** Evolution of rainfall intensity with respect to the return period in Abidjan

Mathematical adapting of the first group of curves (figure 3) leads to formulas of exponential (“power”) functions. Such variable dynamics leads to a simulation by means of the common formula given by Réménéras in 1986:

\[ i_M = a(t - \alpha)^{-b} \]  \hspace{1cm} (5)

Thus, when \( \alpha \) takes the value 0, then equation (5) becomes the Montana formula:

\[ i_M = a(T)t^{-b(T)} \]  \hspace{1cm} (6)

Adjusting the distribution to the trend curves on the Excel spreadsheet makes it possible to determine the values of Montana’s parameters \( a(T) \) and \( b(T) \) (table 2).
Table 2. Montana’s model Parameters a(T) and b(T) for the Abidjan area

<table>
<thead>
<tr>
<th>Return period</th>
<th>a(T)</th>
<th>b(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 years</td>
<td>1492</td>
<td>0.60</td>
</tr>
<tr>
<td>50 years</td>
<td>1334</td>
<td>0.59</td>
</tr>
<tr>
<td>25 years</td>
<td>1181</td>
<td>0.58</td>
</tr>
<tr>
<td>10 years</td>
<td>982</td>
<td>0.57</td>
</tr>
<tr>
<td>5 years</td>
<td>835</td>
<td>0.57</td>
</tr>
<tr>
<td>2 years</td>
<td>628</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Mathematical adapting of the second group of curves (figure 4) has to do with logarithmic (Napierian) functions expressed as:

\[ i_M = a \ln(T) + b \]  \hspace{1cm} (7)

In addition, a comparative study of the two formulas reveals that the Napierian model explicitly uses the return period \( T \) in its expression while the Montana’s model rather uses the downpour duration \( t \). This relevant remark led to searching for an overlapping model which takes into account the return period as well as the downpour duration. This led to expressing the Napierian formula constant \( "a" \) and \( "b" \) with respect to the downpour duration \( "t" \), by means of the Excel spreadsheet once again. This gives:

\[ a = k_t w \]  \hspace{1cm} (8)  \hspace{1cm} and \hspace{1cm} \[ b = \varphi t \]  \hspace{1cm} (9)  

And we deduce from these, that:

\[ i_M = k_t \omega \ln(T) + \varphi t \nu \]  \hspace{1cm} (10)

The values of the parameters of this new model are contained in the table 3 below.

Table 3. Parameters of the overlapping formula

<table>
<thead>
<tr>
<th>Duration (mn)</th>
<th>[10-15]</th>
<th>[15-30]</th>
<th>[30-45]</th>
<th>[45-60]</th>
<th>[60-90]</th>
<th>[90-120]</th>
<th>[120-180]</th>
<th>[180-240]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>68</td>
<td>486</td>
<td>75</td>
<td>178</td>
<td>260</td>
<td>310</td>
<td>405</td>
<td>484</td>
</tr>
<tr>
<td>( \omega )</td>
<td>-0.15</td>
<td>-0.88</td>
<td>-0.329</td>
<td>-0.55</td>
<td>-0.65</td>
<td>-0.69</td>
<td>-0.74</td>
<td>-0.78</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>386</td>
<td>254</td>
<td>344.67</td>
<td>573</td>
<td>839</td>
<td>903</td>
<td>1430</td>
<td>1510</td>
</tr>
<tr>
<td>( \nu )</td>
<td>-0.48</td>
<td>-0.33</td>
<td>-0.425</td>
<td>-0.55</td>
<td>-0.65</td>
<td>-0.66</td>
<td>-0.76</td>
<td>-0.77</td>
</tr>
</tbody>
</table>

DISCUSSION

The originality of this work lies in fact that values characteristic and largely representative of the area of study were taken into account. The advantages of Caquot’s model which includes Montana’s model have already been discussed elsewhere (Réménéiras, 1972; Lémoine and Cruette, 1974; Cruette, 1975; Ikounga, 1976 in Afouda, 1978; Desbordes, 1987). The following discussion concerns the relevance of the results in comparison to the methodology and data used. Firstly, it can be noted that the parameters \( a(T) \) and \( b(T) \) of Montana’s model, were estimated according to a methodology involving data specific to the area of study. It is therefore evident that these estimates differ from those generally used in urban drainage in the Abidjan area. A comparison between the values of \( a(T) \) and \( b(T) \) obtained within the framework of this work and those generally used in calculations relating to urban drainage (table 4), reveals that the adaptation in use underestimates rainfall intensity in the Abidjan area. In actual fact, this current adaptation was realised on the basis of data collected over short observation periods, whereas such an approach is not satisfactory from the representative point of view (OMM, 1983) and does not take into account climatic variability in the area.

Moreover, desiring a more effective rainfall dynamics evaluation in the Abidjan area led to working out a new rainfall model. This new formula (equation 10) best simulates rainfall dynamics in the Abidjan area.
Table 4. Comparison of the parameters a(T) and b(T) of Montana’s model

<table>
<thead>
<tr>
<th>Return period</th>
<th>Parameters currently used in Abidjan area</th>
<th>Optimized parameters for Abidjan area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a(T)</td>
<td>b(T)</td>
</tr>
<tr>
<td>50 years</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>25 years</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10 years</td>
<td>460</td>
<td>-0.37</td>
</tr>
<tr>
<td>5 years</td>
<td>418</td>
<td>-0.37</td>
</tr>
<tr>
<td>2 years</td>
<td>365</td>
<td>-0.37</td>
</tr>
<tr>
<td>1 year</td>
<td>310</td>
<td>-0.37</td>
</tr>
</tbody>
</table>

Actually, it was designed from involving two important factors of the pluvial phenomenon: the duration (t) and the return period (T) and imply four parameters allowing it to give reliably account of the rainfall dynamics. This model thus, simulates the rainfall with a very good correlation, very close to 1 (table 5).

Table 5. Coefficients of correlation values of different return periods.

<table>
<thead>
<tr>
<th>Return period (T)</th>
<th>Coefficient of determination $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 years</td>
<td>0.9842</td>
</tr>
<tr>
<td>5 years</td>
<td>0.9855</td>
</tr>
<tr>
<td>10 years</td>
<td>0.9866</td>
</tr>
<tr>
<td>25 years</td>
<td>0.9882</td>
</tr>
<tr>
<td>50 years</td>
<td>0.9893</td>
</tr>
<tr>
<td>100 years</td>
<td>0.9903</td>
</tr>
</tbody>
</table>

CONCLUSION

Water is vital for life and for many human activities (Barrett et al, 1998). The problem of converting rainfall which falls onto a sloppy basin into drainage flow rate, measured at its outlet, continues to be at the center of the concerns of the hydrologists when it comes to urban drainage management. In African cities and particularly in humid tropical environment, Montana (rainfall) and Caquot (rate-of-flow) models commonly used are very often faced with constraints when it comes to adapting to tropical realities. Thus, floods and erosions continue to considerably affect the population and the urban economy. In this study, climatic variability in Abidjan area has been studied. This investigation reveals that Abidjan region is submitted to an important fluctuation of rainfall. Montana’s model parameters were then optimized for the Abidjan area. This optimization which was done on the basis of statistical processing involving rainfall data collected after a long series of observations, made it possible to realize that the current adaptation of Montana’s model in use in the Abidjan area is not satisfactory and underestimates intensity rainfall. In addition, a new overlapping model which best simulates the dynamics of downpours in Abidjan was designed and its parameters optimised. Though apparently theoretical, the proposed model is inspired by observations carried out on two important parameters of the pluviometric phenomenon; in particular the duration (t) of downpour and its return period (T). Although it is of course necessary to await the results of its implementation to evaluate it, the discussion already indicates that this model is relevant since it links two formulas which are already considered to be satisfactory. A discussion on the implementation of these results and effects on urban drainage in Abidjan would be interesting.
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